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PROPORTION CODES OF NON-RELIGIOUS TRADITIONAL ARCHITECTURE OF EUROPEAN ORIGIN
Triangulature, Quadrature, Quinture:
DRAFT LIST OF THE ANGLES OF REGULATING LINES.

Background Information to the INTBAU essay "J. Langhein, Proportion and Traditional Architecture" (2005), Chap. 6 (www.intbau.org/archive/essay10.htm)

The list given below refers to a lining up of angles of *Regulating Lines* (RL), that are suited to establish *mesh systems of grids* (as shown in the plates 1-12 that followed /attached this post) which I tested during more than two decades in my research on proportion theory and proportion analyzations of farmhouses, townhouses and high domestic architecture of most parts of Europe, East Coast states of the US (including some areas of Turkey and Japan). The research is not finished yet, and will be presented in a two-volume textbook around the year 2014.

The *RL angles* are derived from the most elementary geometry, i.e., those of the regular *polygons with 3* (= equilateral triangle, including the hexagon, dodecagon, their tessellations etc.), *4* (= square, octagon, isosceles in square, Pythagoras Theorem, many constellations & tessellations of squares) and *5 angles* (pentagon, decagon, Golden Mean /Golden Section, Fibonacci ratios) on a circle. These polygons combine the circle (non-linear geometry) with basic and regular linear geometrical figures, and can easily drawn without study of maths and geometry. These basic geometric figures enabled early builders to execute and design any construction in foreseeable, systematic way, that can easily be repeated, modified and verified according the regional and temporal design patterns. Taking the apprenticeship education systems of Middle Ages into account, their rules could easily be taught step by step, always combined with practical work of highly qualified craftsmanship.

In terms of mathematics, the unifying potential of simple polygonal geometry is strongly enhanced by the fact, that most derived configurations comply with symmetry groups of rotational symmetry, and often additionally with laws of topological math.).

After having prepared several thousand proportion analyzations of traditional architecture of Europe and America first without, later by means of CAAD software, I dare to state that approximately 99% of measured drawings I worked on (particularly those elevations of traditional architecture of the three classes [TVA, TSHA, THA] identified in my Intbau essay [footnotes 1-3]) can rationally "submitted" to proportional analyses by means of such "grid meshes" comply with a number of angles indexed below.

Of course, it makes sense also to apply regulating lines in different ways, as the literature shows, or to proportionate buildings in other ways. Thus I never understand my solutions as excluding any other solutions. *Proportion research should be valued in probability ratios*, and are not always contradicting themselves, as often stated in critical literature. I feel that my solutions have often more or less high degree of probability in regard to gestalt pragnance and to regional or temporal frequency distributions. In Switzerland, I prepared more than 100 proportion analyzations that comply with a few modifications of the Quadrature.

(As argued in my essay, proportions in architecture differ from proportions of all other classes visual objects and fields, because only architecture is predominantly shaped by orthogonal features.)

The literature on proportion – listed in my bibliographic database (with keywords, abstracts etc., 51.000 entries, see proportions.de) – seems still to lack a synoptical research work on proportions and design of the three main classes of domestic architecture, although the existence of many important publications. The majority of publications concentrates on religious and other "extraordinary architecture" (like pyramids, unique buildings, etc.) that have no strong significance to domestic architecture of the majority of people.

The "reliability" of the "*table of proportion codes*" is based on their repeated occurrence in architectural objects in all three categories of traditional architecture. The first category of objects, called in my essay TVA (Traditional Vernacular Architecture), appear according to dominate the frequency distributions in the specific regions, remaining often almost unchanged for often many centuries despite the change of styles of high art and high architecture, e.g. the Quadrature type architecture are almost predominant in the TVA and TSHA of Alemannic Switzerland, additionally also in Alemannic areas of Austria (Vorarlberg) and Germany (Oberschwaben, Black Forest); but there is almost no distrubtion in the adherent ancient settlement areas (called in German Kulturgeographie "Altsiedellandschaften") of the Alemannic Alsace (Upper Rhine); the frequency distribution of the Quadrature can be found throughout whole in Europe e.g. in Slavic and ancient Slavic-settled areas, Frisian areas, and in Scotland.

The reliability of proportion analyzations should be viewed in terms of probability ratios. Generally speaking, the *probability ratio* that a "well designed object" or "object with gestalt pragnance" has no systemized formgiving source (or originator of shaping) is very low. In other words, it approaches the degree of total improbability. *Thus, proportion research should be established on probability ratio as a generalized framework of understanding*. Any criticism of my approach should take this into consideration.

The *written documentation* of elementary geometry can be traced back at least to PLATO, who was quoted by VITRUVIUS (e.g. in Book IX, 4). P. FRANKL (The Gothic, Princeton 1960, chap. VI) expressed the view that the oral and secrecy-keeping traditions of Medieval builders (particularly master masons) in regard to application of polygonal geometry in architectural design, "suppressed" a continuity of written documentation. The album or portfolio of Villard de HONNECOURT (dated 1225-1235) and later Mathis RORICZER's famous booklets of 1486 disclosed some essential elements of this practical geometry. Since Renaissance, this practical geometry became constitutional parts of almost every treatise on architecture, crafts, arts, geometry/arithmetic etc. Already Indian, East-Asian and Islamic treatises of Antiquity and Middle Ages gave comprehensive instructions into this practical geometry. My bibliographical database currently contains 7.500 entries of and on such pre-industrial treatises. My textbook will explicitly report on this issue.

The proportion codes are therefore categorized to the traditional categories of "*Triangulature*" – the RL angles derived from the equilateral triangle -, the "*Quadrature*" (derived from the square and octagon-related angles) and "Quinture" (derived from the pentagon & decagon, including the Golden triangle, which seems the by far most representative *Quinture* or Golden Section proportion in architecture (= isosceles triangles in *M:m rectangle* and *m:M rectangle*). The Golden Triangles can be found in many Egyptian and Mesoamerican pyramids. In many regions of Germany and South Europe, the Golden Triangles seem to be a predominant proportion code in traditional architecture.

This explains" the analytical background of proportion analyzations of the images shown in the pdf plates (figures 3 to 7) of my INTBAU Essay 10 (www.intbau.org/essay10.htm). These angles transfer the basic algorithmic order of elementary and regular geometry into 2D surfaces (facades) and 3D bodies of architecture. Proportion research should principally understood to make only probability statements on "configurational facts" of objects. These facts should be based on interpretations, statements, that always will remain more or less probable theses.

All listed angles can be determined by relatively geometrical constructions with compass and ruler only (thus, e.g., repeated on construction sites of traditional architecture).

ANGLES OF REGULATING LINES OF PROPORTIONS DERIVED FROM POLYGONAL GEOMETRY

TRIANGULATURE

			DIAG ONAL IN RECTANG LE 1:3			**Angle tg alpha =x/x-2V3**		
I-1a	**60°**	60	*I-3a*	18°26'6°	18,435		3:√3	
I-1b	**50°**	50	*I-3b*	9°13'3"	9,2175	*I-6°*		21,051700
I-1c	**40°**	40	*I-3c*	71°33'54"	71,565	*I-6b*		23,413224
I-1d	**30°**	30	*I-3d*	35°46'57'"	35,78250	*I-6c*		24,791281
I-1e	**20°**	20	*I-3e*	36°52'12"	36,87000	*I-6d*		25,69335070
I-1f	**70°**	70	*I-3f*	54°13'2"	54,21750			Etc
I-1g	**80°**	80						
I-1h	75°	75	**Angles of tg alpha =x/x-1:Ö3**					
I-1i	65°	65	3:2√3	49°6'23,78"	4 9,10660535			
I-1j	55°	55	*I.4°a/b*	40°53'36,22"	40,89339453	**Länge:Höhe glstg. Dreieck)**		
I-1k	45°	45	4:3√3	52°24'39,28"	52,4109091	*I-7°*	49°6'24	49,10667
I-1l	35°	35	*I-4c/d*	37°35'20,72	37,5808091	*I-7b*	24°33'12"	24,55333
I-1m	25°	25	5:4√3	54°10'56,91'	54,1824757	*I-7c*	81°47'22"	81,86667
I-1n	15°	15	*I-4e/f*	35°49'3,09'	35,8175243	*I-7d*	40°53'36"	40,89333
I-1°	10°	10	6:5√3	55°17'6"	55,2849969	*I-7e*	20°26'48"	20,4466700
I-1p	57,5°	57,5	*I-4g/h*	34°42'54"	34,7150031	*I-7f*	69°33'12"	69,5533300
I-1r	52,5°	52,5	1:√3/		20,4466600	*I-7g*	37°5'12"	37,0866700
I-1s	47,5	47,5	7:6√3	56°2'12,36"	56,0367656	*I-7h*	52°54'48"	52,9133300
I-1t	42,5°	42,5	*I-4j/k*	33°57'47,64"	33,96323441	*I-7i*	26°27'24"	26,4566670
I-1u	37,5°	37,5	8:7√3	56°34'55,27"	56,5820181	*I-7j*	74°10'24"	74,1733300
I-1v	32,5°	32,5	*I-4l/m*	33°25'4,73"	33,4179810	*I-7k*	15°49'36"	15,8266600
I-1w	27,5	27,5	9:8√3	56°59'27,91"	56,99108705	*I-7i*	22°38'15"	22,6375000
I-1x	22,5°	22,5	*I-4n/p*	33°0'32,09"	33,00891295	*I-7j*	52°54'48"	52,7133300
I-1y	17,5°	17,5	10:9√3	57°19'11,34"	57,31981602	*I-7k*	43°10'23,2'	43,17311
I-1z	12,5°	12,5	*I-4q/r*	32°40'48,66"	32,68018398		etc	Etc
	ANG LE RATIOS		11/10√3	57°34'51,49	57,8078763			
I-2a	**35°15'52"**	**35,26439**	*I-4s/t*	32°25'8,51	32,1903012			
I-2b	**54°44'8"**	**54,73556**						
I-2c	**17°37'56"**	**17,63222**						
I-2d	27°22'4"	27,36778						
I-2e	70°31'44"	70,52889						

QUADRATURE

II-1a	**45°**		**RECTANG LE**			**G OTHIC TRIANG LE**		
				0,5:√2				
II-1b	**22,5°**	*157,5*				(von DRACH TRIANGLE		
II-1c	11,25°		*II-2i*	18°26'6'	18,435	im Achtort mit		67,5°, 45° etc
II-1d	16,875°		*II-2j*	71°33'54"	71,565			
II-1e	33,75°	*146,25*	*II-2k*	43°18'60"	43,3138889	**KULMBACH TRIANG LE**		
II-1f	56,25°	*123,75*	*II-2l*	46°41'10"	46,686111	(L.R. Spitzenpfeil)		
II-1g	67,5°			23°20'35'	23,343056		°33'55'	57,56528
In the rectangle 1:√2			i.e. Isocele Triangle within a Square			*II-4b*	54°52'11'	54,86972
vgl. I-2-			*II-3a*	**63°26'6"**	**63,435**	*II-4c*	32°43'48'	32,73

2 /= V3!

II-2a	35°15'52"	35,26439	*II-3b*	**53°7'48'**	**53,13**	*II-4d*	35°7'10"	35,13028
II-2b	54°44'8"	54,73556	*II-3c*	**31°43'3"**	**31,7175**	*II-4e*	32°26'4"	32,43444
II-2c	17°37'56"	17,63222	*II-3d*	**26°33'54"**	**26,565**			
II-2d	27°37'18"	27,36778	*II-3e*	36°52'12"	36,87			
II-2e	70°31'44"	70,52889	*II-3f*	**41°48'37"**	**41,810278**			
II-2f	19°28'16"	18,47111	*II-3g*	24°5'41"	24,09472			
II-2g	39°13'53"	39,23139	*II-3h*	**48°11'23"**	**48,18972**			
II-2h	50°46'7"	50,76861	*II-3i*	83°37'14"	83,62056	**RECTANG LE 1:4 (Tow ers)**		
RECTAG NLE 0,5:Ö2			*II-3j*	58°16'57"	58,2825			
II-2i	18°26'6" tg)	18,435	*II-3k*	73°44'23"	73,73972	*II-5a*	75°57'50'	75,96389
II-2j	71°33'54"	71,565	*II-3l*	**48°53'50"**	**48,89722**	*II-5b*	16°2'10"	16,03611
			II-3m	**41°6'10"**	**41,10278**			
TWO SQUARES ANNEXED			*II-3n*		61,85875			
and other Square-based geometrical Figures			etc.		eTc.			

are not indicated in this diagram,
although they can may have played a
considerable role in large building complexes
(particularly in religious architecture)

QUINTURE

III-1a	54°	54°	III-2d	**19°5'11"**	**19,08639**	*III-4f*	32°57'9"	32,9525
III-1b	36°	36°	III-2e	76°20'43"	76,34528	*III-4g*	12°2'51"	12,0475
III-1c	27°	27°	III-2f	64°5'11"	64,08639	*III-4h*	42°23'31"	42,391944
III-1d	18°	18°	III-2g	57°57'25"	57,95694		(= tg 1/V5:V6)	
III-1e	72°	72°	III-2h	32°2'35"	32,043056	*III-4i*	47°36'29"	47,608056
III-1f	63°	63°	III-2i	31°43'3"	31,7175	*III-4j*	21°11'46"	21,196111
							WH	
III-1g	45°	45°				*III-4k*	23°48'14"	23,803889
							WH	
III-1h	108°	108°				*III-4l*	44°24'44"	44,12222
III-1i	13,5°	13,5°	**ANG LE FUNCTIONS Minor 0,318...**					
III-1j	22,5°	22,5°						
III-1k	31,5°	31,5°	*III-3a*	22°27'20"	22,455556	**RECHTECK 1:5(/25) (Tow ers)**		
III-1l	40,5°	40,5°	*III-3b*	67°32'40"	67,54445	III-5a	11°18'36"	11,31
III-1m	49,5°	49,5°	*III-3c*	20°54'19"	20,905278	III-5b	78°41'24"	78,29
III-1n	58,5°	58,5°	*III-3d*	69°5'41"	69,094722			
III-1o	67,5°	67,5°	*III-3e*	44°54'40"	44,91111	**ROOT of 5, Angle Functions**		
III-1p	76,5°	76,5°	*III-3f*	11°13'20	11,22222			
				WH				
III-1q	85,5°	85,5°	*III-3g*	33°46'20"	33,772222	*III-6a*	26°33'54"	26,565
				WH				
						III-6b	63°26'6"	63,435
G OLDEN TRIANG LE			**RECTANG LE 1:Ö5 od. 1:Ö5*5**			*III-6c*	24°5'41'	24,097222
(Isocele Trianglen a m:M or M Rectangle*			*III-4a*	65.°54'19"	65,905278	*III-6d*	65°54'19'	65,905278
			III-4b	24°5'41"	24,094722	*III-6e*	13°16'57"	13,2825
III-2a	**51°49'38"**	**51,82722**	*III-4c*	22°12'28"	22,207778	*III-6f*	31°43'3"	31,7175
III-2b	**38°10'22"**	**38,17278**	*III-4d*	67°47'32"	67,792222	*III-6g*	24°5'41"	24,094722
III-2c	**25°54'49"**	**25,91361**	*III-4e*	48°11'23"	48,189722	*III-6h*	48°11'23"	48,189722

III-7a	43°24'10"	43,402888
III-7b	46°35'50"	46,597222
III-7c	86°48'19"	86,805278
III-7d	21°42'54"	21,715

III-7e	23°19'55"	23,331944

NON-POLYGONAL ANGLES

PYTHAGORAS THEOREME

corresponds to the Fourth

Ratio 2:3 = FIFTH

2/3-1a	41°48'37,13"	41,81031489
2/3-1b	48°11'22,87"	48,18968511
2/3-1c	33°41'24,24"	33,69006753
2/3-1d	56'18'35,76''	56,30993247

Ratio 4:5 = THIRD Major

4/5-1a	53°7'48,37"	53,13010235
4/5-1b	36°52'11,63"	36,86989765
4/5-1c	38°39'35,31"	38,65980826
4/5-1d	51°20'24,69"	51,34019174

MUSIKALICAL RATIOS

Ratio 5/6 = THIRD Minor

5/6-1a	56°26'33,68"	56,44269024
5/6-1b	33°33'26,32	33,55730760
5/6-1c	39°48'20,06	39,80557109
5/6-1d	50°11'39,94"	50,19442891

Ratio 3:4 = FOURTH

3/4-1a	48°35'25,26"	48,5903779
3/4-1b	41°24'34,64"	41,4096221
3/4-1c	36°52'11,63''	36,8698977
3/4-1d	53°7'48,37"	53,13010235

Ratio 3/5 Sixth Major

3/5-1a	36°52'11,63''	36,86989765
3/5-1b	53°7'48,37"	53,13010235
3/5-1c	30°57'49,52''	30,96375653
3/5-1d	59°2'10,48"	59,03624347

Ratio 5/8 Sixth Minor

5/8-1a	38°40'55,87''	38,68218745
5/8-1b	51°19'4,13"	51,31781255
5/8-1c	32°0'19,38"	32,00538321
5/8-1d	57°59'40,62''	57,99461679

Ratio 1:2 = OCTAVE

1/2-1a	26°33°54,18"	26,56505118
1/2-1b	63°26'5,82"	63,43494882
1/2-1c	53°6'44"	53,11210024
1/2-1d	60	cos
1/2-1e	30	sin

Notes:

This list relates mainly to domestic architecture of the three types TVA, TSHA and THA, but not to religious architecture (with a few exception of Baroque churches in Brazil and Mexico).

By means of a mesh (grid) of RL, the geometrical order (proportions) of whole of facades (2D surfaces) or architectural body become systematically analysable. These proportion research carried out since the early 80s, took primarily traditional vernacular architecture into consideration, excluding almost all religious architecture (although I studied as completely as possible all literature also on it for the establishment of my bibliographic database).

This knowledge can immediately applied in your CAAD software. I add plate of Juan B. de Caramuel y Lobkowicz (1606-1682), because it demonstrates also *tessellations of polygons, being a huge reservoir of interesting affine angles for proportion systems.*

* Grid are systems of modulary repeating coordinating lines (H. ACHTEN et at., 2003).

Aus: Juan CARAMUEL, Arquitectura civil recta y oblicua
Spanien 1678